Maple

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A Maple egy matematikai formula-manipulációs (vagy számítógép-algebrai) rendszer, amelyben nem csak numerikusan, hanem formális változókkal is lehet műveleteket végezni.

Itt rövid általános ismertetőt, ízelítőt adunk és bemutatjuk a legfontosabb számelméleti parancsokat. Lásd a mellékelt Maple munkafájlokat is.

További irodalom:

1. Molnárka Győző és társai, A Maple V és alkalmazásai, Springer Hungarica Kiadó, 1996.

2. Klincsik Mihály, Maróti György, Maple 8 tételben, Novadat Kiadó, 1995.

3. Kovács Zoltán, Rövid bevezetés a Maple használatába, 2004,
http://zeus.nyf.hu/~kovacsz/tutorial.pdf

1. Általános ismertető, ízelítő

```
> # Valos szamok, muveletek
> 3+2; 3-2; # osszeadas, kivonas
```

$$5$$

$$1$$

```
> 2*3; 12/4; # szorzas, osztas
```

$$6$$

$$3$$

```
> 2^4; 2**4; # hatvanyozas
```

$$16$$

$$16$$

```
> sqrt(9); # gyokvonas
```

$$3$$

```
> 1.234; 0.4563; # tizedes tort megadasa
```

$$1.234$$

$$.4563$$

```
> 5!; # faktorialis
```

$$120$$

```
> ifactor(2400); # egesz szam primfelbontasa
```

$$(2)^5\ (3)\ (5)^2$$

```
> Pi, exp(1), gamma, infinity; # pi, e, Euler-allando, vegtelen
```

$$\pi,\ e,\ \gamma,\ \infty$$

```
> pi:=evalf(Pi); e:=evalf(exp(1)); Euler:=evalf(gamma); # kiertekeles
```

$$\pi := 3.141592654$$

$$e := 2.718281828$$

$$Euler := .5772156649$$

```
> Digits:=30; # pontossag
```

$$Digits := 30$$

```
> evalf(Pi); # kiertekeles
```

$$3.141592653589793238462643383279$$

```
> convert(1.2345,fraction); # atalakitas racionlis szamma
```

$$\frac{2469}{2000}$$

```
> floor(Pi); # egeszresz
```

$$3$$

```
> frac(2.3467); # tortresz
```

$$.3467$$

```
> Sum(k^2,k=1..10); sum(k^2,k=1..10); # szumma
```

$$\sum_{k=1}^{10} k^2$$

$$385$$

```
> Sum(k^2,k=1..n); sum(k^2,k=1..n); normal(%); # szumma
```

$$\sum_{k=1}^{n} k^2$$

$$\frac{1}{3}(n+1)^3 - \frac{1}{2}(n+1)^2 + \frac{1}{6}n + \frac{1}{6}$$

$$\frac{1}{3}n^3 + \frac{1}{2}n^2 + \frac{1}{6}n$$

```
> Product(1/i,i=1..7); product(1/i,i=1..7); # szorzat
```

$$\prod_{i=1}^{7} \frac{1}{i}$$

$$\frac{1}{5040}$$

```
> Product(1/i,i=1..n); product(1/i,i=1..n); # szorzat
```

$$\prod_{i=1}^{n} \frac{1}{i}$$

$$\frac{1}{\Gamma(n+1)}$$

```
> ln(3), log(3), evalf(ln(3)); # termeszetes logaritmus;
```

$$\ln(3), \ln(3), 1.098612289$$

```
> log[2](5); log[10](5); evalf(%); # mas alapu es 10-es alapu logaritmus
```

$$\frac{\ln(5)}{\ln(2)}$$

$$\frac{\ln(5)}{\ln(10)}$$

$$.6989700041$$

```
> K:=(x-1)^3*(x+2)^4; expand(K); # kifejezes megadasa, kifejtese
```

$$K := (x-1)^3 (x+2)^4$$

$$x^7 + 5x^6 + 3x^5 - 17x^4 - 16x^3 + 24x^2 + 16x - 16$$

```
> factor(K); # szorzatta alakitas
```

$$(x-1)^3 (x+2)^4$$

```
> F:=(a+b)^2+(3*a-4*b)^2+2*a-7; normal(F); expand(F); # kifejezes szamitasa
```

$$F := (a+b)^2 + (3\,a - 4\,b)^2 + 2\,a - 7$$

$$10\,a^2 - 22\,a\,b + 17\,b^2 + 2\,a - 7$$

$$10\,a^2 - 22\,a\,b + 17\,b^2 + 2\,a - 7$$

```
> T(x):=(x^2-3*x+2)/(x^3+4*x-5); simplify(T(x)); # egyszerusites
```

$$\mathrm{T}(x) := \frac{x^2 - 3\,x + 2}{x^3 + 4\,x - 5}$$

$$\frac{x-2}{x^2 + x + 5}$$

```
> # Komplex szamok
> 2+3*I; # komplex szam megadasa
```

$$2 + 3\,I$$

```
> evalc(Re(2+3*I)), Re(2+3*I); # komplex szam valos resze
```

$$2,\ 2$$

```
> Im(2+3*I); # komplex szam kepzetes resze
```

$$3$$

```
> abs(2+3*I); evalf(%); # komplex szam abszolut erteke
```

$$\sqrt{13}$$

$$3.605551275$$

```
> # Halmazok, halmazmuveletek
> A:={0,1,2,3,4}; B:={1,2,5,6}; # halmaz megadasa
```

$$A := \{0,\ 1,\ 2,\ 3,\ 4\}$$

$$B := \{1,\ 2,\ 5,\ 6\}$$

```
> A union B; A intersect B; A minus B; # halmazmuveletek
```

$$\{0,\ 1,\ 2,\ 3,\ 4,\ 5,\ 6\}$$

$$\{1,\ 2\}$$

$$\{0,\ 3,\ 4\}$$

```
> # Sorozatok, listak
> a:=2,3,4,5; sorozat_1:=seq(2^n,n=1..8); sorozat_2:=seq(i!,i={3,4,5,6}); # sorozatok
megadasa
```

$$a := 2,\ 3,\ 4,\ 5$$

$$\mathit{sorozat\_1} := 2,\ 4,\ 8,\ 16,\ 32,\ 64,\ 128,\ 256$$

$$\mathit{sorozat\_2} := 6,\ 24,\ 120,\ 720$$

```
> lista_1:=[x,y,z,t]; lista_2:=[seq(2^n,n=0..8)]; # listak megadasa
```

$$\mathit{lista\_1} := [x,\ y,\ z,\ t]$$

$$\mathit{lista\_2} := [1,\ 2,\ 4,\ 8,\ 16,\ 32,\ 64,\ 128,\ 256]$$

```
> nops(lista_1); nops(lista_2); # lista elemeinek szama
```

$$4$$

$$9$$

```
> # Egyenletek, egyenletrendszerek
> f:=x^2-6*x+5; solve(f,x); # f(x)=0 gyokei
```

$$f := x^2 - 6\,x + 5$$

$$1,\ 5$$

```
> g:=4*x^4+4*x^3-13*x^2-7*x+8; solve(g,x);
```

$$g := 4\,x^4 + 4\,x^3 - 13\,x^2 - 7\,x + 8$$

$$-\frac{1}{4} + \frac{1}{4}\sqrt{29 + 4\sqrt{17}},\ -\frac{1}{4} - \frac{1}{4}\sqrt{29 + 4\sqrt{17}},\ -\frac{1}{4} + \frac{1}{4}\sqrt{29 - 4\sqrt{17}},\ -\frac{1}{4} - \frac{1}{4}\sqrt{29 - 4\sqrt{17}}$$

```
> fsolve(g,x); # gyokok kozelito ertekei
```

$$-1.936201769,\ -1.134151341,\ .6341513409,\ 1.436201769$$

```
> e_1:=2*x+3*y-4*z=3; e_2:=3*x+4*y+2*z=5; e_3:=x-2*y+7*z=-2;
```

$$e\_1 := 2\,x + 3\,y - 4\,z = 3$$

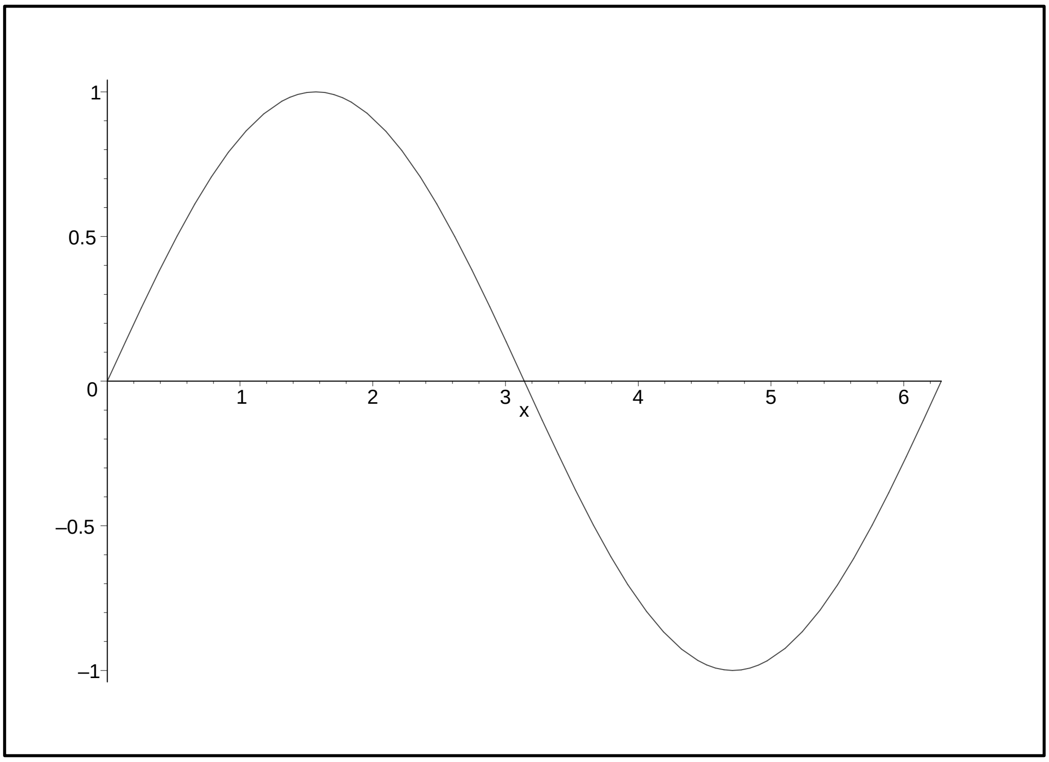
$$e\_2 := 3\,x + 4\,y + 2\,z = 5$$

$$e\_3 := x - 2\,y + 7\,z = -2$$

```
> solve({e_1,e_2,e_3},{x,y,z}); # egyenletrendszer megoldasa
```

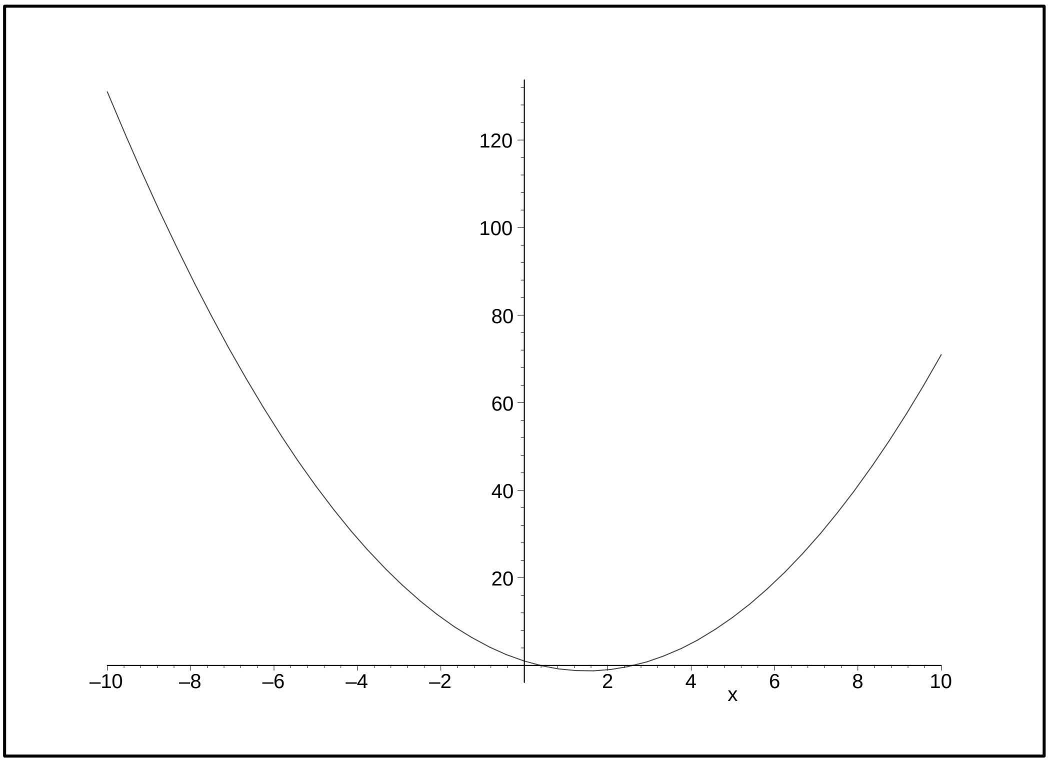
$$\{z = \frac{7}{47},\ y = \frac{65}{47},\ x = \frac{-13}{47}\}$$

```
> # Grafikonok
> plot(sin(x),x=0..2*Pi); # fuggveny grafikonja
```



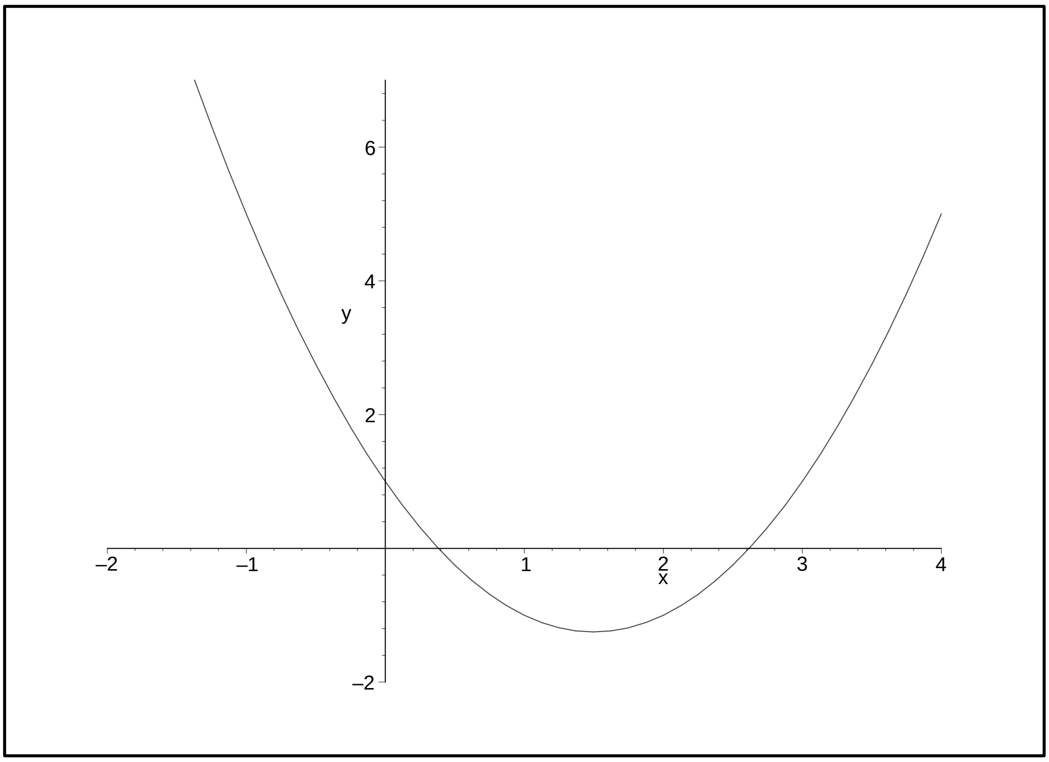

```
> h:=x^2-3*x+1; plot(h,x);
```

$$h := x^2 - 3\,x + 1$$

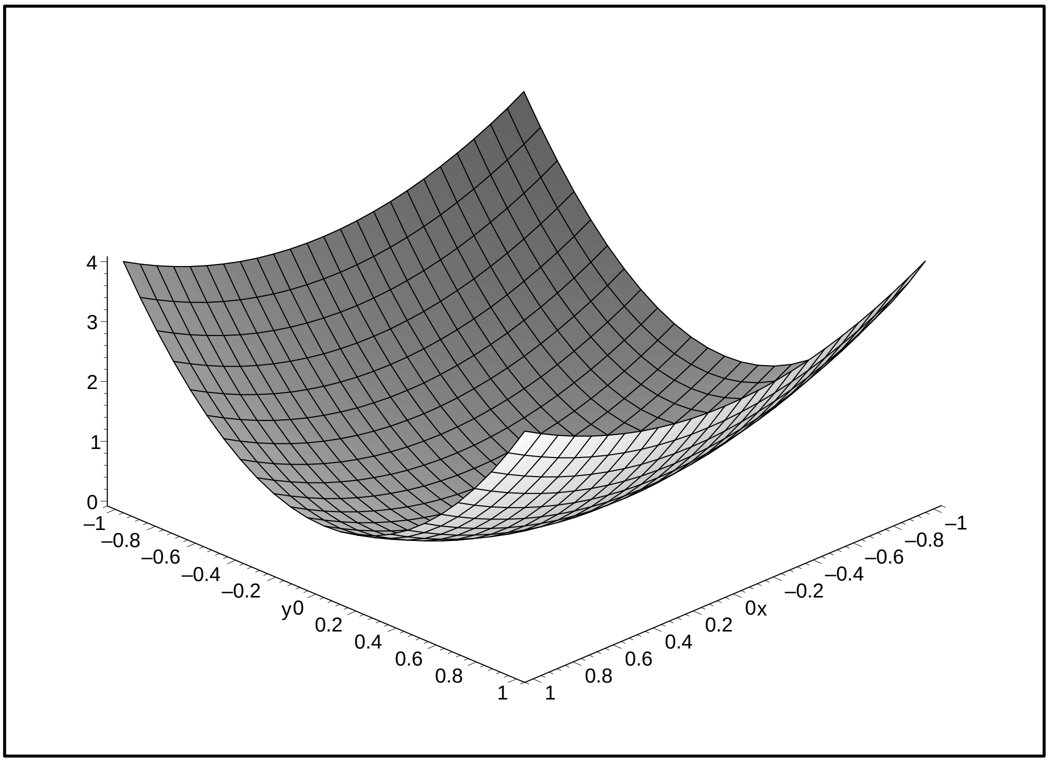



```
> h:=x^2-3*x+1; plot(h,x=-2..4,y=-2..7);
```

$$h := x^2 - 3\,x + 1$$




```
> plot3d(x^2+3*y^2,x=-1..1,y=-1..1);
```




```
> # Mas
> f:=x^3+2*x+6; diff(f,x); int(f,x); int(f,x=-2..4); # fuggveny derivaltja, integralja
```

$$f := x^3 + 2\,x + 6$$

$$3\,x^2 + 2$$

$$\frac{1}{4}\,x^4 + x^2 + 6\,x$$

$$108$$

```
> A:=matrix(3,3,[[1,0,2],[3,4,-1],[7,-3,0]]); B:=matrix(3,3,[[0,1,2],[-1,8,0],[2,-1,3]]);
```

$$A := \begin{bmatrix} 1 & 0 & 2 \\ 3 & 4 & -1 \\ 7 & -3 & 0 \end{bmatrix}$$

$$B := \begin{bmatrix} 0 & 1 & 2 \\ -1 & 8 & 0 \\ 2 & -1 & 3 \end{bmatrix}$$

```
> with(linalg):  evalm(A+B); evalm(2*A-3*B); # Linearis algebra csomag, matrixmuveletek
```

$$\begin{bmatrix} 1 & 1 & 4 \\ 2 & 12 & -1 \\ 9 & -4 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -3 & -2 \\ 9 & -16 & -2 \\ 8 & -3 & -9 \end{bmatrix}$$

```
> evalm(A&*B); # matrix-szorzas
```

$$\begin{bmatrix} 4 & -1 & 8 \\ -6 & 36 & 3 \\ 3 & -17 & 14 \end{bmatrix}$$

```
> with(numtheory): tau(240); sigma(240); phi(240); # Szamelmelet csomag, szam osztoinak
szama, osszege, Euler-fuggveny
```

$$20$$

$$744$$

$$64$$

2. Számelmélet

```
> # Szamelmelet
> with(numtheory): # Szamelmelet csomag
> irem(23,4,'q'); q; # maradekos osztas maradeka es q=hanyadosa
```

$$3$$

$$5$$

```
> iquo(23,4,'r'); r; # maradekos osztas hanyadosa es r=maradeka
```

$$5$$

$$3$$

```
> igcd(38,78); # legnagyobb kozos oszto (lnko)
```

$$2$$

```
> igcd(5,27,32,14);
```

$$1$$

```
> ilcm(35,67); ilcm(12,27,38); # legkisebb kozos tobbszoros (lkkt)
```

$$2345$$

$$2052$$

```
> igcdex(1165,275,'x','y'); x, y; # lnko az euklideszi algoritmussal es x,y megadasa ugy,
hogy (a,b)=ax+by
```

$$5$$

$$17, -72$$

```
> ifactor(123456700); # primtenyezokre bontas
```

$$(2)^2\ (5)^2\ (127)\ (9721)$$

```
> 5^4 mod 7; # maradek (mod n)
```

$$2$$

```
> 2^(2^5)+1 mod 641;
```

$$0$$

```
> p_i:=ithprime(600); #i-edik prim
```

$$p\_i := 4409$$

```
> primek:=seq(ithprime(n),n=1..10);
```

$$primek := 2, 3, 5, 7, 11, 13, 17, 19, 23, 29$$

```
> primeksorozata:=seq([n,ithprime(n)],n=1..10);
```

$$primeksorozata := [1, 2], [2, 3], [3, 5], [4, 7], [5, 11], [6, 13], [7, 17], [8, 19], [9, 23], [10, 29]$$

```
> isprime(145), isprime(173); # prim-e?
```

$$false, true$$

```
> primteszt:=seq([n,isprime(n)],n=1..10);
```

$$primteszt := [1, false], [2, true], [3, true], [4, false], [5, true], [6, false], [7, true], [8, false], [9, false], [10, false]$$

```
> prevprime(1000), nextprime(1000); # n elotti, ill.  utani prim
```

$$997, 1009$$

```
> issqrfree(30), issqrfree(50); # negyzetmentes-e?
```

$$true, false$$

```
> a:=seq([n,issqrfree(n)],n=1..10);
```

$$a := [1, true], [2, true], [3, true], [4, false], [5, true], [6, true], [7, true], [8, false], [9, false], [10, true]$$

```
> divisors(12); # osztok halmaza
```

$$\{1, 2, 3, 4, 6, 12\}$$

```
> seq([n,divisors(n)],n=1..10);
```

$$[1, \{1\}], [2, \{1, 2\}], [3, \{1, 3\}], [4, \{1, 2, 4\}], [5, \{1, 5\}], [6, \{1, 2, 3, 6\}], [7, \{1, 7\}], [8, \{1, 2, 4, 8\}], [9, \{1, 3, 9\}], [10, \{1, 2, 5, 10\}]$$

```
> tau(1000); # osztok szama
```

$$16$$

```
> seq([n,tau(n)],n=1..10);
```

$$[1, 1], [2, 2], [3, 2], [4, 3], [5, 2], [6, 4], [7, 2], [8, 4], [9, 3], [10, 4]$$

```
> seq(tau(n),n=1..20);
```

$$1, 2, 2, 3, 2, 4, 2, 4, 3, 4, 2, 6, 2, 4, 4, 5, 2, 6, 2, 6$$

```
> sigma(1000); # osztok osszege
```

$$2340$$

```
> seq([n,sigma(n)],n=1..10);
```

$$[1, 1], [2, 3], [3, 4], [4, 7], [5, 6], [6, 12], [7, 8], [8, 15], [9, 13], [10, 18]$$

```
> seq(sigma(n),n=1..20);
```

$$1, 3, 4, 7, 6, 12, 8, 15, 13, 18, 12, 28, 14, 24, 24, 31, 18, 39, 20, 42$$

```
> phi(1000); # Euler-fuggveny
```

$$400$$

```
> seq([n,phi(n)],n=1..10);
```

$$[1, 1], [2, 1], [3, 2], [4, 2], [5, 4], [6, 2], [7, 6], [8, 4], [9, 6], [10, 4]$$

```
> seq(phi(n),n=1..10);
```

$$1, 1, 2, 2, 4, 2, 6, 4, 6, 4$$

```
> seq([n,mobius(n)],n=1..10); # Mobius-fuggveny
```

$$[1, 1], [2, -1], [3, -1], [4, 0], [5, -1], [6, 1], [7, -1], [8, 0], [9, 0], [10, 1]$$

```
> seq(mobius(n),n=1..10);
```

$$1, -1, -1, 0, -1, 1, -1, 0, 0, 1$$

```
> fermat(n); # Fermat-szamok
```

$$2^{(2^n)} + 1$$

```
> fermat(5);
```

$$4294967297$$

```
> fermat(5,'w');
```

$$4294967297$$

```
> w;
```

$$\mathit{it\ is\ completely\ factored}\ ,\ (\ (2)^7\ \ (5) + 1)\ \ (\ (2)^7\ \ (3)\ \ (17449) + 1)$$

```
> mersenne(5); mersenne(11); # a 2^n-1 erteke, ha az prim, Mersenne-szam
```

$$31$$

$$\mathit{false}$$

```
> mersenne([4]); # az n-edik Mersenne-prim
```

$$127$$

```
> legendre(5,13), legendre(4,13); # Legendre-szimbolum
```

$$-1, 1$$

```
> lambda(13), lambda(200); # Carmichael-fuggveny
```

$$12, 20$$

```
> bigomega(40); # Omega-fuggveny (primhatvanyosztok szama)
```

$$4$$

```
> mcombine(7,4,11,5); # Kinai maradektetel, az mcombine(m,a,n,b) megad egy olyan x
egesz szamot, amelyre x=a (mod m), x=b (mod n)
```

$$60$$

```
> mcombine(5,4,15,2);
```

$$\mathit{FAIL}$$

```
> order(3,10); # order(a,m) az a rendje (mod m)
```

$$4$$

```
> mlog(11,3,17); # index:  mlog(a,g,m) megadja az ind_g(a) mod n szamot
```

$$7$$

```
> mlog(11,4,17);
```

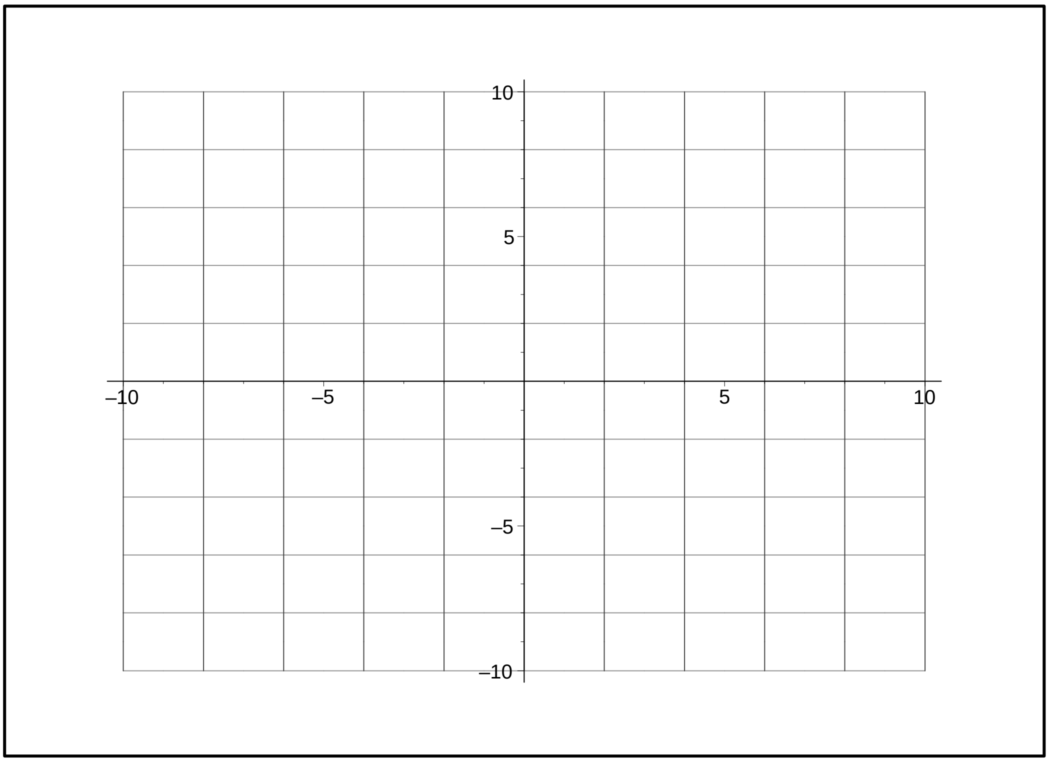
$$\mathit{FAIL}$$

```
> primroot(4,17); # primroot(a,m) a legkisebb a-nal negyobb primitiv gyok (mod m)
```

$$5$$

3. Gauss-egészek

```
> # Gauss-egeszek
> with(GaussInt): # Gauss-egeszek csomag
> with(plots): conformal(z, z=-10-10*I..10+10*I); # a komplex sik
```




```
> GInorm(2+13*I); # norma
```

$$173$$

```
> GInearest(-2.34+6.87*I), GInearest((2+8*I)/(3-7*I)); # egy komplex szamhoz legkozelebbi
Gauss-egesz
```

$$-2+7\,I,\ -1+I$$

```
> GInormal(3-2*I); # az elso negyedbeli asszocialt
```

$$2+3\,I$$

```
> GIgcd(-345+515*I,1574+368*I); # lnko
```

$$41+117\,I$$

```
> GIgcdex(-345+515*I,1574+368*I,'u','v'), u, v; # lnko(z,t) az euklideszi algoritmussal es
olyan u,v megadasa, amelyekre (z,t)=uz+vt
```

$$41+117\,I,\ 5-2\,I,\ -2\,I$$

```
> GIquo(6+12*I,-8+4*I,'r'), r; # maradekos osztas hanyadosa es r=maradeka
```

$$-I,\ 2+4\,I$$

```
> GIrem(6+12*I,-8+4*I,'q'), q; # maradekos osztas maradeka es q=hanyadosa
```

$$2+4\,I,\ -I$$

```
> GIlcm(1+2*I,-3+5*I,14+3*I); # lkkt
```

$$61+57\,I$$

```
> GIprime(751), GIprime(4+9*I), GIprime(-3+4*I); # Gauss-prim teszt
```

$$\mathit{true},\ \mathit{true},\ \mathit{false}$$

```
> GIfactor(12101-15295*I); # primtenyezokre bontas
```

$$(-1)\ (1+I)\ (1+4\,I)\ (5+8\,I)\ (9+16\,I)\ (-7-18\,I)$$

```
> expand(%); # visszaalakitas
```

$$12101-15295\,I$$

```
> GIsieve(10); # GIsieve(m) megadja azokat az x+yI Gauss-primeket, amelyek normaja <= m^2
es 0<y<=x
```

$$[14,[1+I,\,1+2\,I,\,3,\,2+3\,I,\,1+4\,I,\,2+5\,I,\,1+6\,I,\,4+5\,I,\,7,\,2+7\,I,\,5+6\,I,\,3+8\,I,\,5+8\,I,\,4+9\,I]]$$

```
> GIdivisor(161+240*I); # az elso negyedbeli osztok
```

$$\{1,\,4+I,\,161+240\,I,\,52+47\,I,\,15+8\,I\}$$

```
> GInodiv(10-2*I); # a nemasszocialt osztok szama
```

$$8$$

```
> GIphi(-21+5*I); # GIphi(m) egy mod m redukalt maradekrendszer elemeinek a szama
```

$$232$$